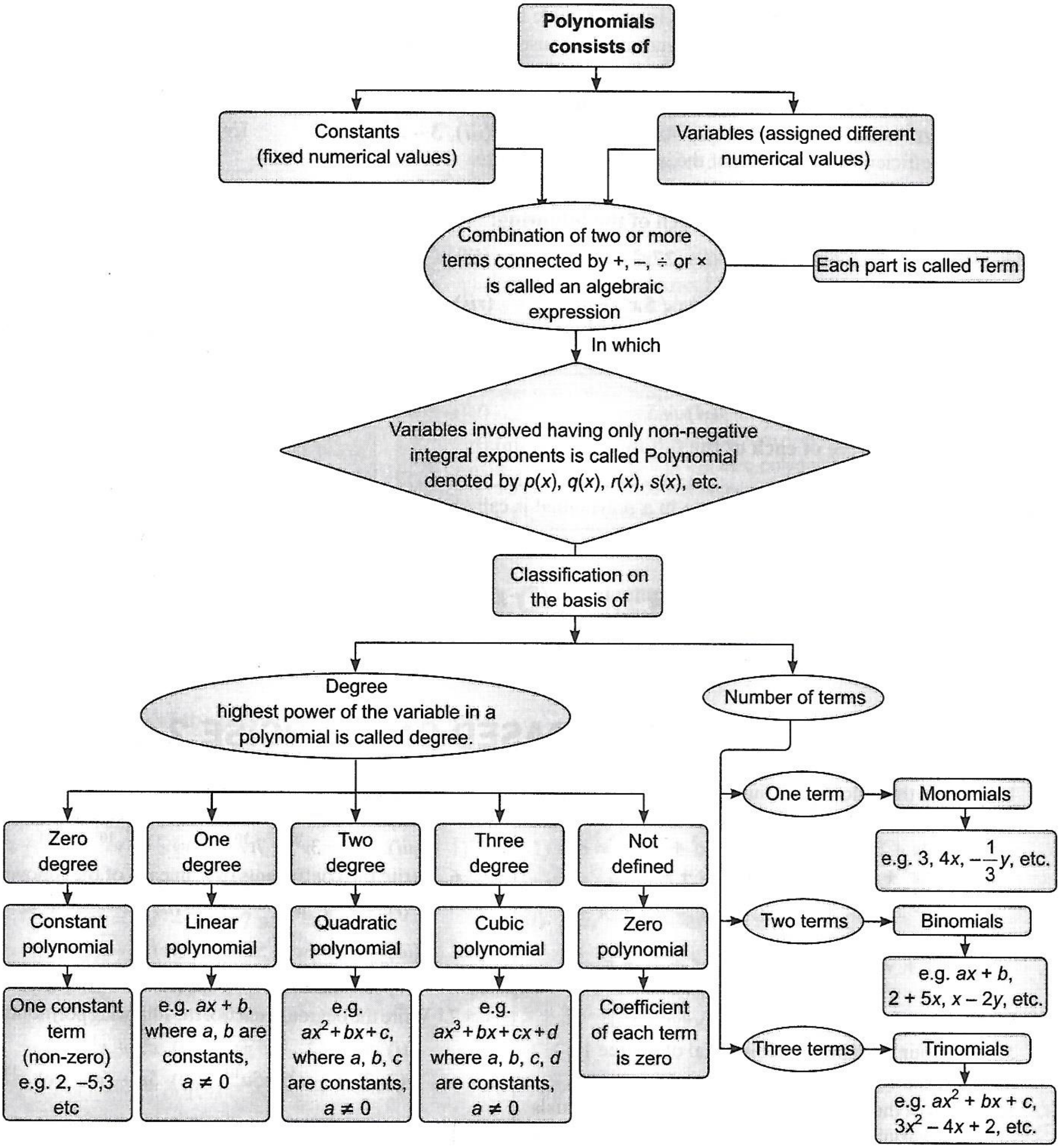
2 POLYNOMIALS

Introduction to Polynomials and Polynomials in one Variable



SOLVED QUESTIONS BASED ON EXERCISE 2.1

Very Short Answer Type Questions [1 Mark]

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $3y^2 + 4y$ (ii) $2t^2 + 5$ (iii) $y + \frac{1}{y}$ (iv) $\sqrt{x} + 4$

(v) $-x^3 + 3x^2 + 5x - 3$ (vi) $3x^2 - 5x + 8$ (vii) $\sqrt[3]{t} + t^2$ (viii) $4\sqrt{t} + t\sqrt{3}$

Sol. (i), (ii), (v) and (vi) are polynomials in one variable because these expressions contain one variable only.

(iii), (iv), (vii) and (viii) are not polynomials because the exponent of the variable in these expressions are not a whole number.

2. Write the coefficients of x^2 in each of the following:

(i) $ax^2 + bx + c$ (ii) $4x^2 + 5x - 3$ (iii) $3 - 5x^2 + x^3$ (iv) $\sqrt{2}x^2 + 4y + 5$

Sol. The coefficients of x^2 in each of the given polynomials are:

(i) a (ii) 4 (iii) -5 (iv) $\sqrt{2}$

3. Write the coefficients of x^3 in each of the following:

(i) $5x^3 - 6x^2 + 7x - 9$ (ii) $27x^3 - y^3$ (iii) $\frac{2}{5}x^3 - x$ (iv) $2x^3 + 5$

(v) $x^3 + x + 6$ (vi) $\sqrt{5}x^3 + 1$ (vii) $y^3 - 16x^3$ (viii) $3x^4 - 4x^3 - 3x - 5$

Sol. The coefficients of x^3 in the given expressions are:

(i) 5 (ii) 27 (iii) $\frac{2}{5}$ (iv) 2

(v) 1 (vi) $\sqrt{5}$ (vii) -16 (viii) -4

4. Write the degree of each of the following polynomials:

(i) $4x^5 - 3x^4 + 9$ (ii) $3 - 2y^2 + 5y^3 - 2y^8$ (iii) 3 (iv) $4y + 5$

Sol. The highest power of the variable in a polynomial is called the degree of the polynomial. Hence, degree of the given expressions are

(i) 5 (ii) 8 (iii) 0 ($\because 3 = 3x^0$) (iv) 1

5. Write the coefficient of y in the expansion of $(5 - y)^2$. [CBSE 2016]

Sol.

$$(5 - y) = 5^2 - 2 \times 5 \times y + y^2 \quad [\text{Using } (a - b)^2 = a^2 - 2ab + b^2]$$
$$= 25 - 10y + y^2$$

$\therefore$ Required coefficient of $y = -10$

PRACTICE QUESTIONS BASED ON EXERCISE 2.1

1. Classify the following as linear, quadratic and cubic polynomials:
   (i) $4x + 5$ (ii) $2x^2 + 5$
   (iii) $5x^3 + 6x^2$ (iv) $2y + 4$
   (v) $u^2 - \frac{5}{2}u$ (vi) $2t^3 + 4t^2 + 5t + 7$
   (vii) $t + \sqrt{3}$ (viii) $5y^2 + 3y + \pi$
   (ix) $5 - y^2 + y^3$ (x) $4x^3$
2. Give four examples of a linear polynomial.
3. Give four examples of a binomial of degree 15.
4. Write monomials of degree 150.
5. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.
   (i) $x^{15} + x^{10} + y^2$ (ii) $4 - x^2 + 5y^3$
   (iii) $2x^5 + 3y^6 + 7t^{10}$ (iv) $\sqrt{3}y^{10} + 5x^{11} - z^{16}$
6. Write the coefficients of x^2 in each of the following:
   (i) $\frac{\pi}{4}x^2 + 5x^0$ (ii) $9x^3 - 5x^2 + 4x - 6$
   (iii) $-x^3 + 6x^2 - 5x + 3$ (iv) $4\sqrt{2}x^2 + 5$
   (v) $5 - \sqrt{3}x^2 + 4x^3$ (vi) $3 + 7x^2 - x$
7. Write the degree of each of the following polynomials:
   (i) $u + \sqrt{2}$ (ii) $x^{30} + x^{15} + 4$
   (iii) $2 - u - u^3 + 5u^{10}$ (iv) $5x^3 - 6x^2$ (v) x^{150}

Zeroes of a Polynomial

Zeroes (Roots) of a polynomial

A number $x = a$ is called zero of the polynomial $p(x)$, if $p(a) = 0$

Can be obtained by

Hit and trial method

By putting $x = a$ in a given polynomial and check
(i) If $p(a) = 0$, then 'a' is a zero of the given polynomial.
(ii) If $p(a) \neq 0$, then 'a' is not a zero of the given polynomial.

Example:
- Verify whether 3 and 0 are zeroes of the polynomial $2x^2 - 3x$.

Sol. Let $p(x) = 2x^2 - 3x$

Then $p(3) = 2 \times 3^2 - 3 \times 3 = 18 - 9 = 9 \neq 0$

$p(0) = 0 - 0 = 0$

Hence, 0 is a zero of the polynomial $2x^2 - 3x$, but 3 is not.

Solving the polynomial equation by taking $p(x) = 0$

Example: Let $p(x) = 2x + 3$

Its root is given by $p(x) = 0 \Rightarrow 2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

Thus, $x = -\frac{3}{2}$ is a zero of $p(x) = 2x + 3$

Its properties are

- Every linear polynomial has unique zero.
- A non-zero constant polynomial has no zero.
- A polynomial can have more than one zero depending upon its nature.
- Every real number is a zero of the zero polynomial.
- A zero of a polynomial need not be zero.
- 0 may be the zero of a polynomial.

SOLVED QUESTIONS BASED ON EXERCISE 2.2

Very Short Answer Type Questions [1 Mark]

1. Find the value of the polynomial $y^2 - 5y + 6$ at

(i) $y = 0$ (ii) $y = -1$

Sol. The value of the polynomial $p(y) = y^2 - 5y + 6$

(i) at $y = 0$ is given by $p(0) = 0 - 5(0) + 6 = 6$

(ii) at $y = -1$ is given by $p(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$

2. Find the value of each of the following polynomials at the indicated value of variable:

(i) $p(y) = 5y^2 - 3y + 7$ at $y = 1, -1$ (ii) $q(x) = 3x^3 - 4x + \sqrt{8}$ at $x = 2$

Sol. (i) $p(y) = 5y^2 - 3y + 7$

$\therefore$ At $y = 1, p(1) = 5(1)^2 - 3(1) + 7 = 5 - 3 + 7 = 9$

and at $y = -1, p(-1) = 5(-1)^2 - 3(-1) + 7 = 5 + 3 + 7 = 15$

(ii) $q(x) = 3x^3 - 4x + \sqrt{8}$

$\therefore$ at $x = 2, q(2) = 3(2)^3 - 4 \times 2 + \sqrt{8} = 24 - 8 + \sqrt{8} = 16 + \sqrt{8}$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = x^3 - 3x^2 + 4x - 12, x = 3$ **(ii) $p(y) = y^4 - 3y^2 + 2y + 1, y = 1$**

Sol. (i) $p(x) = x^3 - 3x^2 + 4x - 12$

$\therefore\ p(3) = 3^3 - 3(3)^2 + 4(3) - 12 = 27 - 27 + 12 - 12 = 0$

So, $x = 3$ is a zero of the polynomial $p(x) = x^3 - 3x^2 + 4x - 12$

(ii) $p(y) = y^4 - 3y^2 + 2y + 1$

$\therefore\ p(1) = 1^4 - 3(1)^2 + 2(1) + 1 = 1 - 3 + 2 + 1 = 1 \neq 0$

So, $y = 1$ is not a zero of the polynomial $p(y) = y^4 - 3y^2 + 2y + 1$

Short Answer Type Questions I [2 Marks]

4. If – 1 is a zero of the polynomial $p(x) = ax^3 - x^2 + x + 4$, then find the value of 'a'.

Sol. If – 1 is a zero of the polynomial $p(x)$, then

$$p(-1) = 0$$

$\Rightarrow\quad a(-1)^3 - (-1)^2 + (-1) + 4 = 0$

$\Rightarrow\quad -a - 1 - 1 + 4 = 0$

$\Rightarrow\quad -a + 2 = 0 \ \Rightarrow\ a = 2$

5. What is the value of polynomial $x^2 + 8x + k$, if – 1 is a zero of the polynomial? [CBSE 201

Sol.

$$p(x) = x^2 + 8x + k$$

If – 1 is a zero of the polynomial $p(x)$, then

$$p(-1) = 0$$

$\Rightarrow\quad (-1)^2\, 8(-1) + k = 0$

$\Rightarrow\quad 1 - 8 + k = 0 \ \Rightarrow k = 7$

Short Answer Type Questions II [3 Marks]

6. If $x = -\frac{1}{3}$ is a zero of a polynomial $p(x) = 27x^3 - ax^2 - x + 3$, then find the value of 'a'. [CBSE 201

Sol. Given:

$$p(x) = 27x^3 - ax^2 - x + 3$$

If $x = -\frac{1}{3}$ is the zero of $p(x)$, then,

$$p\left(-\frac{1}{3}\right) = 0$$

$\Rightarrow\quad 27\left(-\frac{1}{3}\right)^3 - a\left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 3 = 0$

$\Rightarrow\quad -27 \times \frac{1}{27} - \frac{a}{3} + \frac{1}{3} + 3 = 0$

$\Rightarrow\quad \frac{a}{3} = -1 + \frac{10}{3} = \frac{7}{3} \ \Rightarrow\ a = 7$

7. If $f(x) = 5x^2 - 4x + 5$, find $f(1) + f(-1) + f(0)$ [CBSE 2016

Sol.

$$f(x) = 5x^2 - 4x + 5$$

So,

$$f(1) = 5(1)^2 - 4(1) + 5 = 5 - 4 + 5 = 6$$

$$f(-1) = 5(-1)^2 - 4(-1) + 5 = 5 + 4 + 5 = 14$$

$$f(0) = 5(0)^2 - 4.0 + 5 = 0 - 0 + 5 = 5$$

$\therefore\quad f(1) + f(-1) + f(0) = 6 + 14 + 5 = 25$

PRACTICE QUESTIONS BASED ON EXERCISE 2.2

1. Find the value of the polynomial $p(y) = y^2 - 5y + 6$ at
   (i) $y = 2$ (ii) $y = -2$
2. Find the value of each of the following polynomials at indicated value of variables:
   (i) $p(t) = 4t^4 + 5t^3 - t^2 + 8$ at $t = b$
   (ii) $r(x) = 5x^3 - 2x^2 + 3x - 4$ at $x = 1, -1$
3. Verify whether the following are zeroes of the polynomials, indicated against them.
   (i) $r(t) = at + b, t = \frac{-b}{a}$
   (ii) $g(x) = 2x^2 + x - 1, x = \frac{-1}{2}$
   (iii) $p(x) = 2x + 3, x = \frac{-3}{2}$
4. Find the zero of the polynomial in each of the following cases:
   (i) $p(x) = 5x$
   (ii) $p(y) = ay + b, a \neq 0, a, b$ are real numbers
   (iii) $p(x) = -4x + 5$
   (iv) $p(y) = 3y + 1$
5. Find the zero of the polynomial in each of the following cases:
   (i) $p(x) = 3x - 4$
   (ii) $p(x) = 3x + 4$
   (iii) $p(x) = 7$
6. Give zeroes of the polynomial $p(x) = (x - 1)(x - 2)(x - 3)(x - 4)$ **[CBSE 2014]**
7. If $f(x) = x^2 - 4x + 6$, find $f(1) - f(-1)$

Theorems in Polynomials

Theorems in Polynomials

Remainder theorem

If $p(x)$ be any polynomial of degree, $n \geq 1$, and 'a' be any real number. When $p(x)$ is divided by linear polynomial $g(x) = (x - a)$ the remainder is $y(x) = p(a)$ and focuses on finding the remainder even if $p(x)$ is not divided by $g(x)$

Example: Find the remainder when the polynomial $p(y) = y^4 - 2y^2 + 3y - 1$ is divided by $g(y) = y - 1$

Sol. Here, the zero of $y - 1$ is 1. By remainder theorem, the remainder is:

$$p(1) = 1^4 - 2 \times 1^2 + 3 \times 1 - 1$$
$$= 1 - 2 + 3 - 1$$
$$= 1$$

Factor theorem

If $p(x)$ is a polynomial of degree, $n \geq 1$ and 'a' is any real number, then

(i) $x - a$ is a factor of $p(x)$ if $p(a) = 0$ and

(ii) $p(a) = 0$ if $(x - a)$ is a factor of $p(x)$

It focuses on

- Checking operation in division of numbers.
- Finding linear factor of $p(x)$.

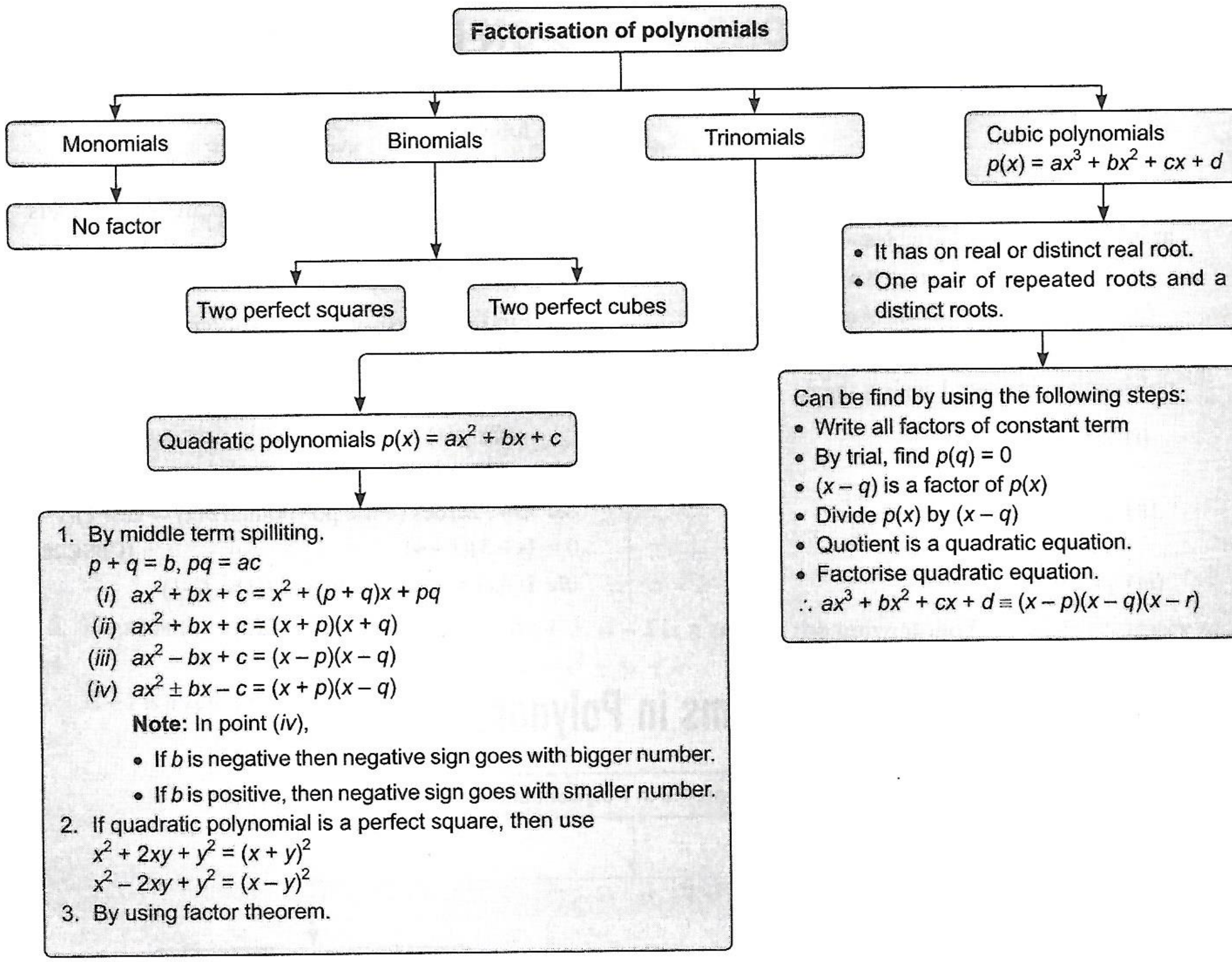
Example: Determine whether $x - 3$ is a factor of polynomial $p(x) = x^3 - 4x^2 - 3x + 18$

Sol. For $x - 3$ to be a factor of $p(x)$, by factor theorem, $p(3)$ should be zero, where 3 is the zero of $x - 3$.

Now, by reminder theorem,

$$p(3) = 3^3 - 4.3^2 - 3.3 + 18$$
$$= 27 - 36 - 9 + 18$$
$$= 45 - 45 = 0$$

Hence, by Factor Theorem, $x - 3$ is a factor of the given polynomial $p(x)$.



SOLVED QUESTIONS BASED ON EXERCISES 2.3 AND 2.4

Short Answer Type Questions I [2 Marks]

1. Find the remainder when $4x^3 - 3x^2 + 4x - 2$ is divided by

(i) $x - 1$ (ii) $x - 2$

Sol. (i) Let $p(x) = 4x^3 - 3x^2 + 4x - 2$ and $g(x) = x - 1$. Zero of $g(x)$ is 1

By the remainder theorem, the remainder is, $p(1) = 4 \times 1^3 - 3 \times 1^2 + 4 \times 1 - 2$
$= 4 - 3 + 4 - 2 = 8 - 5 = 3$

Hence, remainder is 3.

(ii) Let, $p(x) = 4x^3 - 3x^2 + 4x - 2$ and $g(x) = x - 2$

Here, zero of $g(x) = 2$ [$g(x) = 0 \Rightarrow x = 2$]

Using remainder theorem, when $p(x)$ is divided by $g(x)$, we get the remainder $p(2)$

$p(2) = 4(2)^3 - 3(2)^2 + 4(2) - 2 = 32 - 12 + 8 - 2 = 40 - 14 = 26$

Hence, remainder is 26.

2. By remainder theorem, find the remainder when $p(y)$ is divided by $g(y)$

(i) $p(y) = 4y^3 - 12y^2 + 5y - 4$ and $g(y) = 2y - 1$

(ii) $p(y) = y^3 - 4y^2 - 2y + 6$ and $g(y) = 1 - \frac{3}{4}y$.

Sol. (*i*) Hero, zero of $g(y)$ is $\frac{1}{2}$ ...$[2y - 1 = 0 \Rightarrow y = \frac{1}{2}]$

Using remainder theorem, when $p(y)$ is divided by $g(y)$ the remainder, we get, is $p\left(\frac{1}{2}\right)$

$$\therefore \quad p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) - 4$$

$$= \frac{4}{8} - \frac{12}{4} + \frac{5}{2} - 4$$

$$= \frac{4 - 24 + 20 - 32}{8} = -\frac{32}{8} = -4$$

Hence, required remainder is (-4).

(*ii*) $p(y) = y^3 - 4y^2 - 2y + 6$, and $g(y) = 1 - \frac{3}{4}y$

Here, zero of $g(y)$ is $\frac{4}{3}$ $[1 - \frac{3}{4}y = 0 \Rightarrow y = \frac{4}{3}]$

Using remainder theorem, when $p(y)$ is divided by $g(y)$, the remainder, we get is $p\left(\frac{4}{3}\right)$

$$\therefore \quad p\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 4\left(\frac{4}{3}\right)^2 - 2\left(\frac{4}{3}\right) + 6$$

$$= \frac{64}{27} - \frac{64}{9} - \frac{8}{3} + 6$$

$$= \frac{64 - 192 - 72 + 162}{27} = \frac{226 - 264}{27} = -\frac{38}{27}$$

Hence, required remainder is $\left(-\frac{38}{27}\right)$.

3. Check whether $p(x)$ is multiple of $g(x)$ or not.

(*i*) $p(x) = x^3 + 27x^2 - 8x + 18$, $g(x) = x - 1$

(*ii*) $p(x) = 2\sqrt{2}x^3 - 5\sqrt{2}x^2 + 7\sqrt{2}$, $g(x) = x + 1$

Sol. (*i*) $g(x) = x - 1 \quad \therefore$ zero of $g(x)$ is 1

Now, $p(x) = x^3 + 27x^2 - 8x + 18$

$\therefore \quad p(1) = (1)^3 + 27(1)^2 - 8(1) + 18 = 1 + 27 - 8 + 18$

$= 38 \neq 0$

Since $p(1) \neq 0$, so $p(x)$ is not a multiple of $g(x)$.

(*ii*) $g(x) = x + 1 \quad \therefore$ zero of $g(x)$ is -1

Now, $p(x) = 2\sqrt{2}x^3 - 5\sqrt{2}x^2 + 7\sqrt{2}$

$p(-1) = 2\sqrt{2}(-1)^3 - 5\sqrt{2}(-1)^2 + 7\sqrt{2} = -2\sqrt{2} - 5\sqrt{2} + 7\sqrt{2} = 0$

Since $p(1) = 0$, then $p(x)$ is a multiple of $g(x)$.

4. Examine whether $x - 1$ is a factor of the following polynomials:

(*i*) $2x^3 - 5x^2 + x + 2$ **(*ii*) $4x^3 + 5x^2 - 3x + 6$**

Sol. If $x - a$ is a factor of $p(x)$, then by factor theorem, $p(a) = 0$. Here for all expressions, $x = 1$.

(*i*) Let $p(x) = 2x^3 - 5x^2 + x + 2$

$p(1) = 2(1)^3 - 5(1)^2 + 1 + 2 = 2 - 5 + 1 + 2 = 5 - 5 = 0$

Hence, $x - 1$ is a factor of $p(x) = 2x^3 - 5x^2 + x + 2$.

(*ii*) Let $p(x) = 4x^3 + 5x^2 - 3x + 6$

$p(1) = 4(1)^3 + 5(1)^2 - 3(1) + 6 = 4 + 5 - 3 + 6 = 15 - 3 = 12$

$\Rightarrow \quad p(1) = 12 \neq 0$

Hence, $x - 1$ is not a factor $p(x) = 4x^3 + 5x^2 - 3x + 6$.

5. Find the value of k, if $x + k$ is the factor of the polynomials:

(i) $x^3 + kx^2 - 2x + k + 5$ **(ii) $x^4 - k^2x^2 + 3x - 6k$**

Sol. Using factor theorem, if $x + k$ is a factor of $p(x)$ then $p(-k) = 0$. Here for all expressions $x = -k$.

(i) Let $p(x) = x^3 + kx^2 - 2x + k + 5$

$\therefore$ $p(-k) = (-k)^3 + k(-k)^2 - 2(-k) + k + 5 = 0$

$\Rightarrow$ $-k^3 + k^3 + 3k + 5 = 0$

$\Rightarrow$ $k = -\frac{5}{3}$

(ii) Let $p(x) = x^4 - k^2x^2 + 3x - 6k$

$p(-k) = (-k)^4 - k^2.(-k)^2 + 3(-k) - 6k = 0$

$\Rightarrow$ $k^4 - k^4 - 3k - 6k = 0$

$\Rightarrow$ $-9k = 0 \Rightarrow k = 0$

Short Answer Type Questions II [3 Marks]

6. Let R_1 and R_2 are the remainders when polynomial $f(x) = 4x^3 + 3x^2 - 12ax - 5$ and $g(x) = 2x^3 + ax^2$ $6x - 2$ are divided by $(x - 1)$ and $(x - 2)$ respectively. If $3R_1 + R_2 - 28 = 0$, find the value of a.

[CBSE 2015, HOT

Sol. Given: $f(x) = 4x^3 + 3x^2 - 12ax - 5$ and

$g(x) = 2x^3 + ax^2 - 6x - 2$

Now, R_1 = remainder when $f(x)$ is divided by $x - 1$

$\Rightarrow$ $R_1 = f(1)$

$\Rightarrow$ $R_1 = 4(1)^3 + 3(1)^2 - 12a(-1) - 5$

$= 4 + 3 + 12a - 5 = 12a + 2$

And R_2 = remainder when $g(x)$ is divided by $x - 2$

$\Rightarrow$ $R_2 = g(2)$

$= 2(2)^3 + a(2)^2 - 6(2) - 2$

$= 16 + 4a - 12 - 2 = 4a + 2$

Substituting the value of R_1 and R_2 in $3R_1 + R_2 - 28 = 0$, we get

$3(12a + 2) + 4a + 2 - 28 = 0$

$36a + 6 + 4a - 26 = 0$

$40a = 20$

$a = \frac{20}{40} = \frac{1}{2}$

7. Divide $3y^4 - 8y^3 - y^2 - 5y - 5$ by $y - 3$ and find the quotient and the remainder. **[CBSE 201**

Sol.

$$\begin{array}{r|l} & 3y^3 + y^2 + 2y + 1 \\ y - 3 & 3y^4 - 8y^3 - y^2 - 5y - 5 \\ & 3y^4 - 9y^3 \\ & \underline{- \quad +} \\ & y^3 - y^2 \\ & y^3 - 3y^2 \\ & \underline{- \quad +} \\ & 2y^2 - 5y \\ & 2y^2 - 6y \\ & \underline{- \quad +} \\ & y - 5 \\ & y - 3 \\ & \underline{- \quad +} \\ & -2 \end{array}$$

$\therefore$ Quotient $= 3y^3 + y^2 + 2y + 1$

Remainder $= -2$

Long Answer Type Questions [4 Marks]

8. Find the value of a and b so that $x + 1$ and $x - 1$ are factors of $x^4 + ax^3 + 2x^2 - 3x + b$.

Sol. Let $f(x) = x^4 + ax^3 + 2x^2 - 3x + b$ be the given polynomial and $g(x) = x + 1$, $h(x) = x - 1$

If $g(x)$ is a factor of $f(x)$, then by factor theorem,

$$f(-1) = 0$$

$$\Rightarrow \quad (-1)^4 + a(-1)^3 + 2(-1)^2 - 3(-1) + b = 0$$

$$\Rightarrow \quad 1 - a + 2 + 3 + b = 0$$

$$\Rightarrow \quad -a + b = -6 \qquad ...(i)$$

If $h(x)$ be a factor of $f(x)$, then, again by factor theorem,

$$f(1) = 0$$

$$\Rightarrow \quad 1^4 + a(1)^4 + 2(1)^2 - 3(1) + b = 0$$

$$\Rightarrow \quad 1 + a + 2 - 3 + b = 0$$

$$\Rightarrow \quad a + b = 0 \qquad ...(ii)$$

Adding (i) and (ii), we get

$$2b = -6 \text{ or } b = -3$$

From (ii), we have $\quad a - 3 = 0 \Rightarrow a = 3$

Hence, required value of a and b are 3 and – 3 respectively.

9. State factor theorem. Using this theorem, factorise $x^3 - 3x^2 - x + 3$. **[CBSE 2011]**

Sol. **Factor theorem:** If $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number, then

(i) $x = a$ is a factor of $p(x)$, if $p(a) = 0$ and

(ii) $p(a) = 0$ if $(x - a)$ is a factor of $p(x)$

Let $\quad p(x) = x^3 - 3x^2 - x + 3 \qquad ...(i)$

Factors of 3 are ± 1, and ± 3.

By trial method,

$$p(1) = 1^3 - 3(1)^2 - 1 + 3 = 1 - 3 - 1 + 3 = 0$$

Therefore, by factor theorem, $(x - 1)$ is a factor of $p(x)$.

Similarly, $\quad p(-1) = (-1)^3 - 3(-1)^2 - (-1) + 3$

$$= -1 - 3 + 1 + 3 = 0$$

So, $(x + 1)$ is also a factor of $p(x)$ and $p(3) = (3)^3 - 3(3)^2 - 3 + 3 = 27 - 27 - 3 + 3 = 0$

So, $(x - 3)$ is also a factor of $p(x)$.

Since $p(x)$ is a polynomial of degree 3, so it cannot have more than three linear factors.

Let $\quad p(x) = k(x - 1)(x + 1)(x - 3) \qquad ...(ii)$

$$\Rightarrow \quad x^3 - 3x^2 - x + 3 = k(x - 1)(x + 1)(x - 3)$$

Putting $x = 0$ both sides, we get

$$3 = k(-1)(1)(-3) = 3k$$

$$\Rightarrow \quad k = 1$$

Putting $k = 1$ in equation (ii), we get

$$f(x) = (x - 1)(x + 1)(x - 3)$$

10. Divide polynomial $p(x) = 3x^4 + 4x^3 + 4x^2 - 8x + 1$ by $q(x) = 3x + 1$. Also, find what should be added to $p(x)$ so that it is completely divisible by $q(x)$.

Sol.

$$\begin{array}{r|l} & x^3 + x^2 + x - 3 \\ \hline 3x+1 & 3x^4 + 4x^3 + 4x^2 - 8x + 1 \\ & \underline{\overset{-}{3x^4} + \overset{-}{x^3}} \\ & \quad 3x^3 + 4x^2 \\ & \quad \underline{\overset{-}{3x^3} + \overset{-}{x^2}} \\ & \qquad 3x^2 - 8x \\ & \qquad \underline{\overset{-}{3x^2} + \overset{-}{x}} \\ & \qquad\quad -9x + 1 \\ & \qquad\quad \underline{\overset{+}{-9x} - \overset{+}{3}} \\ & \qquad\qquad 4 \end{array}$$

$\therefore$ Quotient $= x^3 + x^2 + x - 3 = g(x)$

Remainder $= 4 = r(x)$

Now, we know that

$$p(x) = (x - a)g(x) + r(x)$$

$\Rightarrow$ For $r(x)$ to be zero, then we must added -4 to $p(x)$ so that $p(x)$ must be exactly divisible by $g(x)$.

PRACTICE QUESTIONS BASED ON EXERCISES 2.3 AND 2.4

1. Find the remainder when $4x^3 - 3x^2 + 4x - 2$ is divided by
   (*i*) $x + 2$ (*ii*) $x + \frac{1}{2}$
2. Examine whether $x - 1$ is a factor of the following polynomials:
   (*i*) $4x^3 + 3x^2 - 4x - 3$
   (*ii*) $x^3 - 3x^2 - 9x + 5$
3. Find the value of k, if $x + k$ is factor of the polynomials:
   (*i*) $x^3 - (k^2 - 1)x + 3$
   (*ii*) $-4x^3 + 4x^2 + 4kx - k$
4. Show that whether $g(x)$ is a factor of the polynomials:
   (*i*) $f(x) = 2\sqrt{2}x^2 - 5\sqrt{x} - 3\sqrt{2}$; $g(x) = x - 2$
   (*ii*) $f(x) = 2x^4 - 12x^3 + 18x + 14$; $g(x) = x - 2$
   (*iii*) $f(x) = 7x^2 - 2\sqrt{8}x - 6$; $g(x) = x - \sqrt{2}$
5. Find the value of a and b so that polynomial $p(x) = x^3 - 3x^2 - ax + b$ has $(x + 1)$ and $(x - 5)$ as factors.
6. Factorise:
   (*i*) $y^3 - 7y + 6$
   (*ii*) $2x^3 - 5x^2 - 19x + 42$
   (*iii*) $3z^3 - 4z^2 - 12z + 16$
   (*iv*) $x^3 - 6x^2 + 11x - 6$ **[CBSE 2016]**
7. By actual division, find the quotient and remainder when $3x^4 - 4x^3 - 3x - 1$ is divided by $x + 1$. **[CBSE 2016]**
8. If the polynomial $f(x) = px^3 + 4x^2 + 3x - 4$ and $g(x) = x^3 - 4 + p$ are divided by $(x - 3)$, then the remainder in each case is the same. Find the value of p. **[CBSE 2011]**
9. Find the value of m and n so that the polynomial $f(x) = x^3 - 6x^2 + mx - n$ is exactly divisible by $(x - 1)$ as well as $(x - 2)$. **[CBSE 2011]**
10. The polynomial $p(x) = 2x^3 + kx^2 - 3x + 5$ and $q(x) = x^3 + 2x^3 - x + k$, when divided by $(x - 2)$ leave the same remainder r_1 and r_2 respectively. Find the value of k, if $r_1 - r_2 = 0$ **[CBSE 2016]**

Algebraic Identities

Identity I:	$(x + y)^2 = x^2 + 2xy + y^2$
Identity II:	$(x - y)^2 = x^2 - 2xy + y^2$
Identity III:	$x^2 - y^2 = (x + y)(x - y)$
Identity IV:	$(x + a)(x + b) = x^2 + (a + b)x + ab$
Identity V:	$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
Identity VI:	$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
Identity VII:	$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
	$= x^3 - 3x^2y + 3xy^2 - y^3$
Identity VIII:	$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

SOLVED QUESTIONS BASED ON EXERCISE 2.5

Short Answer Type Questions I [2 Marks]

1. Find the following products by using suitable identities.

(*i*) $(2x + 5)(2x - 5)$ (*ii*) $(x^2 + 4)(x^2 - 4)$

(*iii*) $(4 + 5x)(4 + 5x)$ (*iv*) $(3x + 4y)(3x - 8y)$

Sol. (*i*) $(2x + 5)(2x - 5) = (2x)^2 - 5^2$

$= 4x^2 - 25$ [Using $(a + b)(a - b) = a^2 - b^2$]

(*ii*) Using identity $(a + b)(a - b) = a^2 - b^2$

We have, $(x^2 + 4)(x^2 - 4) = (x^2)^2 - (4)^2$

$= x^4 - 16$

(*iii*) Using identity $(a + b)^2 = a^2 + 2ab + b^2$

We have, $(4 + 5x)(4 + 5x) = (4 + 5x)^2 = 4^2 + 2(4)(5x) + (5x)^2$

$= 16 + 40x + 25x^2$

$= 25x^4 + 40x + 16$

(*iv*) Using identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

We have, $(3x + 4y)(3x - 8y) = (3x)^2 + [(4y) + (-8y)](3x) + (4y)(-8y)$

$= 9x^2 + (-4y)(3x) - 32y^2$

$= 9x^2 - 12xy - 32y^2.$

2. Evaluate the following products by using suitable identities.

(*i*) 104×105 (*ii*) 102×98

(*iii*) $116 \times 116 - 106 \times 106$

Sol. (*i*) $104 \times 105 = (100 + 4) \times (100 + 5)$

$= 100^2 + (4 + 5) \times 100 + 4 \times 5$

[Using $(x + a)(x + b) = [(x^2 + (a + b)x + ab]$

$= 10000 + 900 + 20 = 10920$

(*ii*) $102 \times 98 = (100 + 2)(100 - 2)$

$= (100)^2 - (2)^2$ [Using $(x + y)(x - y) = x^2 - y^2$]

$= 10000 - 4 = 9996$

(*iii*) $116 \times 116 - 106 \times 106 = (116)^2 - (106)^2$

$= (116 + 106)(116 - 106)$ [Using $a^2 - b^2 = (a + b)(a - b)$]

$= 222 \times 10 = 2220$

3. Factorise the following using suitable identities:

(*i*) $x^2 - y^2 + 2x + 1$ (*ii*) $9a^2 - 4b^2 - 6a + 1$ (*iii*) $a^4 - 16b^4$

Sol. (*i*) $x^2 - y^2 + 2x + 1 = (x^2 + 2x + 1) - y^2$

$= (x + 1)^2 - y^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]

$= (x + 1 + y)(x + 1 - y)$ $(a^2 - b^2) = (a + b)(a - b)$]

$= (x + y + 1)(x - y + 1)$

(*ii*) $9a^2 - 4b^2 - 6a + 1 = (9a^2 - 6a + 1) - 4b^2$

$= [(3a)^2 - 2(3a)(1) + (1)^2] - (2b)^2$ [Using $x^2 - 2xy + y^2 = (x - y)^2$]

$= (3a - 1)^2 - (2b)^2$

$= (3a - 1 + 2b)(3a - 1 - 2b)$ [Using $x^2 - y^2 = (x + y)(x - y)$]

$= (3a + 2b - 1)(3a - 2b - 1)$

(*iii*) $a^4 - 16b^4 = (a^2)^2 - (4b^2)^2$

$= (a^2 + 4b^2)(a^2 - 4b^2)$ [Using $x^2 - y^2 = (x + y)(x - y)$]

$= (a^2 + 4b^2)[(a)^2 - (2b)^2]$

$= (a^2 + 4b^2)(a + 2b)(a - 2b)$

4. Factorise the following:

(*i*) $3\sqrt{3}a^3 + 8b^3 - 27c^3 + 18\sqrt{3}\,abc$ (*ii*) $a^3 - 8b^3 + 1 + 6ab$

(*iii*) $(a + b)^3 - (a - b)^3$

Sol. (*i*) $3\sqrt{3}a^3 + 8b^3 - 27c^3 + 18\sqrt{3}abc = (\sqrt{3}a)^3 + (2b)^3 + (-3c)^3 - 3(\sqrt{3}a)(2b)(-3c)$

$$= (\sqrt{3}a + 2b - 3c) \times [(\sqrt{3}a)^2 + (2b)^2 + (-3c)^2 - (\sqrt{3}a)(2b) - (2b)(-3c) - (-3c)(\sqrt{3}a)]$$

[By Using, $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$]

$$= (\sqrt{3}a + 2b - 3c)(3a^2 + 4b^2 + 9c^2 - 2\sqrt{3}ab + 6bc + 3\sqrt{3}ca)$$

(*ii*) $a^3 - 8b^3 + 1 + 6ab = a^3 + (-2b)^3 + (1)^3 - 3(a)(-2b)(1)$

$$= (a - 2b + 1)[(a)^2 + (-2b)^2 + (1)^2 - a(-2b) - (-2b)(1) - (a)(1)]$$

$$= (a - 2b + 1)(a^2 + 4b^2 + 1 + 2ab + 2b - a)$$

(*iii*) Using identity: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

We have, $x = a + b$ and $y = a - b$

So, $(a + b)^3 - (a - b)^3 = [(a + b) - (a - b)][(a + b)^2 + (a + b)(a - b) + (a - b)^2]$

$$= (a + b - a + b)(a^2 + b^2 + 2ab + a^2 - b^2 + a^2 + b^2 - 2ab)$$

$$= 2b(3a^2 + b^2)$$

Short Answer Type Questions II [3 Marks]

5. Prove that: $(x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(z + x) = 2(x^3 + y^3 + z^3 - 3xyz)$

Sol. We know

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \quad ...(i)$$

Let $a = x + y, b = y + z$ and $c = z + x$

So, $a + b + c = (x + y) + (y + z) + (z + x)$

$$= 2x + 2y + 2z = 2(x + y + z)$$

and $a^2 + b^2 + c^2 - ab - bc - ca = (x + y)^2 + (y + z)^2 + (z + x)^2 - (x + y)(y + z) - (y + z)(z + x) - (z + x)(x + y)$

$$= x^2 + y^2 + 2xy + y^2 + z^2 + 2yz + z^2 + x^2 + 2zx - xy - xz - y^2 - yz - yz - xy - z^2 - zx - zx - zy - x^2 - xy$$

$$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx - 3xy - 3yz - 3zx$$

$$= x^2 + y^2 + z^2 - xy - yz - zx$$

Substituting the value in (*i*), we get

$(x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(z - x)$

$$= 2(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= 2(x^3 + y^3 + z^3 - 3xyz)$$

Hence proved.

6. If $2x + 3y = 12$ and $xy = 6$, find the value of $8x^3 + 27y^3$. **[CBSE 2016]**

Sol. We know that

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

$$\Rightarrow \quad x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

Now,

$$8x^3 + 27y^3 = (2x)^3 + (3y)^3$$
$$= (2x+3y)^3 - 3(2x)(3y)(2x+3y)$$
$$= 12^3 - 18 \times 6 \times 12 \qquad [\text{Given } 2x + 3y = 12 \text{ and } xy = 6]$$
$$= 1728 - 1296 = 432$$

Hence,

$$8x^3 + 27y^3 = 432$$

7. If $z^2 + \frac{1}{z^2} = 34$, find the value of $z^3 + \frac{1}{z^3}$ using only the positive value of $z + \frac{1}{z}$. **[CBSE 2016]**

Sol. Using identity

$$(x+y)^2 = x^2 + 2xy + y^2$$

We have,

$$\left(z+\frac{1}{z}\right)^2 = z^2 + 2z\cdot\frac{1}{z} + \frac{1}{z^2}$$

$$\Rightarrow \quad = \left(z^2+\frac{1}{z^2}\right) + 2$$
$$= 34 + 2 = 36$$

$$\Rightarrow \quad z+\frac{1}{z} = \pm\sqrt{36} = \pm 6$$

According to the given condition, we should use only possible value of $z + \frac{1}{z}$

$$\therefore \quad z+\frac{1}{z} = 6$$

Now, again using the identity

$$x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

We have,

$$z^3+\frac{1}{z^3} = \left(z+\frac{1}{z}\right)^3 - 3z\cdot\frac{1}{z}\left(z+\frac{1}{z}\right)$$
$$= 6^3 - 3 \times 6$$
$$= 216 - 18 = 198$$

Long Answer Type Questions [4 Marks]

8. Factorise the following:

(i) $x^2 - \frac{y^2}{9}$ **(ii)** $2x^2 - 7x - 15$ **[CBSE 2016]**

Sol. (i)

$$x^2 - \frac{y^2}{9} = x^2 - \left(\frac{y}{3}\right)^2$$
$$= \left(x-\frac{y}{3}\right)\left(x+\frac{y}{3}\right) \qquad [x^2 - y^2 = (x+y)(x-y)]$$

(ii)

$$2x^2 - 7x - 15 = 2x^2 - 10x + 3x - 15$$
$$= 2x(x-5) + 3(x-5) = (x-5)(2x+3)$$

9. If $ab + bc + ca = 0$, find the value of $\dfrac{1}{a^2 - bc} + \dfrac{1}{b^2 - ca} + \dfrac{1}{c^2 - ab}$. **[CBSE 2016, HOTS]**

Sol. Given: $ab + bc + ca = 0$

$\Rightarrow \quad -ab = bc + ca; -bc = ca + ab$ and $-ca = ab + bc$

Therefore,
$$\frac{1}{a^2-bc}+\frac{1}{b^2-ca}+\frac{1}{c^2-ab} = \frac{1}{a^2+ab+ca}+\frac{1}{b^2+ab+bc}+\frac{1}{c^2+bc+ca}$$
$$= \frac{1}{a(a+b+c)}+\frac{1}{b(b+a+c)}+\frac{1}{c(c+b+a)}$$
$$= \frac{1}{(a+b+c)}\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right]$$
$$= \frac{1}{(a+b+c)}\times\left(\frac{ab+bc+ca}{abc}\right)$$
$$= \frac{0}{abc(a+b+c)} = 0 \qquad \text{as } ab + bc + ca = 0$$

10. It is given that $3a + 2b = 5c$, then find the value of $27a^3 + 8b^3 - 125c^3$ if $abc = 0$. **[CBSE 2016]**

Sol. Given: $3a + 2b = 5c$

Taking cube both sides, we have

$$(3a + 2b)^3 = (5c)^3$$
$$\Rightarrow \quad (3a)^3 + (2b)^3 + 3(3a)(2b)(3a + 2b) = 125c^3 \qquad [\text{Using: } (x + y)^3 = x^3 + y^3 + 3xy(x + y)]$$
$$\Rightarrow \quad 27a^3 + 8b^3 + 18ab\,(3a + 2b) = 125c^3$$
$$\Rightarrow \quad 27a^3 + 8b^3 + 18ab(5c) = 125c^3 \qquad [\because 3a + 2b = 5c]$$
$$\Rightarrow \quad 27a^3 + 8b^3 + 90\,abc = 125c^3$$
$$\Rightarrow \quad 27a^3 + 8b^3 + 90 \times 0 = 125c^3$$
$$\Rightarrow \quad 27a^3 + 8b^3 - 125c^3 = 0 \qquad [\because abc = 0]$$

11. If $a + b + c = 0$, then prove that $\dfrac{(b + c)^2}{3bc} + \dfrac{(c + a)^2}{3ac} + \dfrac{(a + b)^2}{3ab} = 1$. **[CBSE 2014]**

Sol. L.H.S.
$$= \frac{(b+c)^2}{3bc}+\frac{(c+a)^2}{3ac}+\frac{(a+b)^2}{3ab}$$
$$= \frac{b^2+c^2+2bc}{3bc}+\frac{c^2+a^2+2ac}{3ac}+\frac{a^2+b^2+2ab}{3ab} \qquad [\text{Using: } (x + y)^2 = x^2 + y^2 + 2xy]$$
$$= \frac{1}{3abc}[ab^2 + ac^2 + 2abc + bc^2 + ba^2 + 2abc + a^2c + b^2c + 2abc]$$
$$= \frac{1}{3abc}[ab^2 + ac^2 + bc^2 + ba^2 + a^2c + b^2c + 6abc]$$
$$= \frac{1}{3abc}[ab(b + a) + ac(c + a) + bc(c + b) + 6abc]$$
$$= \frac{1}{3abc}[ab(-c) + ac(-b) + bc(-a) + 6abc] \qquad [\because a + b + c = 0]$$
$$= \frac{1}{3abc}[-abc - abc - abc + 6abc]$$
$$= \frac{3abc}{3abc} = 1 = \text{R.H.S.}$$

Hence proved.

PRACTICE QUESTIONS BASED ON EXERCISE 2.5

1. Prove that:
   $(x-y)^3 + (y-z)^3 + (z-x)^3 = 3(x-y)(y-z)(z-x)$
2. Find the value of $x^3 - 27y^3 - 343 - 27xy$, when $x = 3y + 7$
3. If $x + y + z = 6$ and $xy + yz + zx = 12$, then show that: $x^3 + y^3 + z^3 = 3xyz$
4. Find the following products by using suitable identities:
   (i) $(x + 2y)(x^2 - 2xy + 4y^2)$
   (ii) $(\sqrt{x} - \sqrt{y})(\sqrt{x} - \sqrt{y})$
5. Evaluate the following products by using suitable identities only:
   (i) 99^2 (ii) 99^3
   (iii) $1000^3 - 999^3$
6. Factorise the following using suitable identities:
   (i) $x^3 + \frac{3}{2}x^2 + \frac{3}{4}x + \frac{1}{8}$
   (ii) $(x + 3)^2 + p^2 + 2p(x + 3)$
   (iii) $2x^2 + 11x - 21$
   (iv) $\frac{1}{2}y^2 - 3y + 4$
7. Factorise the following:
   (i) $x^3 + 3x^2y + 3xy^2 + y^3 - 125$
   (ii) $8a^3 - 27b^3 - 64c^3 - 64c^3 - 72abc$
8. Simplify: $(x + y + z)^2 - (x - y + z)^2$ **[CBSE 2016]**
9. Using suitable identity, evaluate
   $(-32)^3 + (18)^3 + (14)^3$
10. If $a + b + c = 0$, then prove that
    $a^4 + b^4 + c^4 = 2(b^2c^2 + c^2a^2 + a^2b^2)$
    [CBSE 2016, HOTS]

Value Based Questions

1. Gram panchayat of village Admapur district Gonda U.P. decided to donate a piece of land of area $2(a^2 + b^2 + c^2 - ab - bc - ca)$. One of its part to government hospital, other Anganwadi for the integrated child development services program to combat child hunger and malnutrition and last part to set up a primary basic educational institute for the villagers.
   (i) Find the share of land which can be given to each organization.
   (ii) Which values do you think everyone should learn from this activity?
2. Pooja distributed cuboidal gifts to the children in an orphanage on her birthday. What are the possible expression for the dimensions of the cuboid if the volume is $3kx^2 + 2kx - 5k$. What values of Pooja are depicted here? **[CBSE 2016]**
3. During the time of flood in some parts of Bihar and Odisha, a group of students decided to help them with clothes, food items, medicines, etc. So, this group collected some amount of rupees from the various sources which is represented by $ax^4 + 2x^3 - 3x^2 + bx - 4$. If number of students is represented by $x^2 - 4$, then find the values of a and b. What values are depicted by that group of students?
4. In a restaurant at the time of payment, the owner says that you divide $x^3 - 3x^2 - x + 6$ by $x - 3$ and pay that money. If the owner does not return any balance, then find the amount paid and the remaining balance. If the owner returns the balance, then what values are depicted by him?

INTEGRATED EXERCISE

Very Short Answer Type Questions [1 Mark]

1. Give an example of a polynomial.
2. Find the degree of polynomial $\sqrt{2}$.
3. Find the degree of the polynomial
   $4x^4 + 0x^3 + 0x^5 + 5x + 7$. **[NCERT Exemplar]**
4. Find the degree of the zero polynomial.

5. If $p(x) = x^2 - 2\sqrt{2}x+1$, then find the value of $p(2\sqrt{2})$. **[NCERT Exemplar]**

6. Find the value of the polynomial $5x - 4x^2 + 3$, when $x = -1$. **[NCERT Exemplar]**

7. If $p(x) = x + 3$, then find the value of $p(x) + p(-x)$.

8. Find the zero of the zero polynomial.

9. Find the zero of the polynomial $p(x) = 2x + 5$.

10. Find the zero of the polynomial $2x^2 + 7x - 4$.

Short Answer Type Questions I [2 Marks]

11. If $x^{51} + 51$ is divided by $x + 1$, then find the remainder. **[NCERT Exemplar]**

12. If $x + 1$ is a factor of the polynomial $2x^2 + kx$, then find the value of k. **[NCERT Exemplar]**

13. Check whether $x + 1$ is a factor of the polynomial $x^3 + x^2 + x + 1$. **[NCERT Exemplar]**

14. Find the factor of $(25x^2 - 1) + (1 + 5x)^2$.

15. Find the value of $249^2 - 248^2$.

16. Factorise $4x^2 + 8x + 3$.

17. Find a factor of $(x + y)^3 - (x^3 + y^3)$.

18. Find the coefficient of x in the expansion of $(x + 3)^3$.

19. If $\frac{x}{y}+\frac{y}{x} = -1\ (x, y \neq 0)$, then find the value of $x^3 - y^3$.

20. If $49x^2 - b = \left(7x+\frac{1}{2}\right)\left(7x-\frac{1}{2}\right)$, then find the value of b. **[NCERT Exemplar]**

21. If $a + b + c = 0$, then find the value of $a^3 + b^3 + c^3$.

Short Answer Type Questions II [3 Marks]

22. Find the value of polynomial $3x^3 - 4x^2 + 7x - 5$ when $x = 3$ and when $x = -3$.

23. If $p(x) = x^2 - 4x + 3$ then evaluate $p(2) - p(-1) + p\left(\frac{1}{2}\right)$.

24. Find the zeroes of the polynomial $p(x) = (x - 2)^2 - (x + 2)^2$.

25. Show that
    (*i*) $x + 3$ is a factor of $69 + 11x - x^2 + x^3$
    (*ii*) $2x - 3$ is a factor of $x + 2x^3 - 9x^2 + 12$

26. Show that $p - 1$ is a factor of both $p^{10} - 1$ and $p^{11} - 1$.

27. For what value of m is $x^3 - 2mx^2 + 16$ is divisible by $x + 2$?

28. Find the value of m, so that $2x - 1$ be a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$.

29. If $a + b + c = 9$ and $ab + bc + ca = 26$ find $a^2 + b^2 + c^2$.

30. Factorise the following
    (*i*) $1 - 64a^3 - 12a + 48a^2$
    (*ii*) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$

31. Without finding the cubes, factorise:
    $(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$

32. Find the value of
    (*i*) $x^3 + y^3 - 12xy + 64$ when $x + y = -4$
    (*ii*) $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$

33. Give possible expression for the length and breadth of the rectangle whose area is given by $4a^2 + 4a - 3$.

Long Answer Type Questions [4 Marks]

34. If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$. Find the value of a. **[NCERT Exemplar]**

35. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$ leaves the remainder 19. Find the value of a. Also, find the remainder when $p(x)$ is divided by $x + 2$. **[NCERT Exemplar]**

36. If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, then show that $p = r$.

37. Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.

38. Simplify: $(2x - 5y)^3 - (2x + 5y)^3$

39. Multiply: $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $(-z + x - 2y)$

40. If a, b and c are all non-zero and $a + b + c = 0$, then prove that $\frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = 3$ **[NCERT Exemplar]**

41. If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

42. Prove that:
$(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$
[NCERT Exemplar]

ASSESS YOURSELF

1. Give an example of a cubic polynomial.
2. Find the degree of polynomial $4x^3 + 9x - 6x^5$.
3. Find the coefficient of x^3 in the expression $p(x) = (3x - 4)(x^2 + 5x - 1)$.
4. Find the zeroes of $x^2 - 2x$.
5. Find the value of $346^2 - 345^2$.
6. Classify the following as linear, quadratic, cubic and constant polynomials:
   (*i*) $5x - 2\sqrt{2}$ (*ii*) $9 - 8u^2$
   (*iii*) $8 - x^3 + 2x^2$ (*iv*) 7
7. Find the zero of the polynomial $p(x) = (x - 4)^2 - (x - 6)^2$.
8. By remainder theorem, find the remainder when $p(x) = 4x^3 - 3x^2 + 2x - 5$ is divided by $g(x) = 1 - 2x$.
9. If $y + 1$ is a factor of $ky^3 + y^2 - 2y + 4k - 10$, then find the value of k.
10. Factorise:
   (*i*) $42 - x - x^2$ (*ii*) $16x^2 + 25 - 40x$
11. Factorise: $2x^3 - x^2 + 13x - 6$.
12. Evaluate the following by using suitable identity:
   (*i*) 101×99 (*ii*) 999^3
13. Without actually calculating the cubes, find the value of
   (*i*) $(-14)^3 + 8^3 + 6^3$
   (*ii*) $27^3 + (-14)^3 + (-13)^3$
14. If $3x - 2y = 13$ and $xy = 5$, find the value of $27x^3 - 8y^3$.
15. Simplify: $(p + q)^3 - (p - q)^3 - 6q(p^2 - q^2)$.
16. When $f(y) = y^4 - 4y^3 + 8y^2 - my + n$ is divided by $y + 1$ and $y - 1$, we get remainder as 10 and 16 respectively. Find the remainder if $f(y)$ is divided by $y - 3$.
17. If $x^2 - 1$ is a factor of $px^4 + qx^3 + rx^2 + sx + u$, show that $p + r + u = q + s = 0$.
18. If $x^2 + \frac{1}{x^2} = 66$, find the value of $x^3 - \frac{1}{x^3}$.
19. What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$. So that result is exactly divisible by $x^2 + x - 2$?
20. Using factor theorem, factorise the polynomial $x^4 - x^3 - 7x^2 + x + 6$.